small, and this circumstance appears to favour the view that a slow surface-reduction is in progress."

XII. "On the Values of the Integrals $\int_0^1 Q_n, Q_{n'}, d\mu$, Q_n, $Q_{n'}$ being Laplace's Coefficients of the Orders n, n', with an application to the Theory of Radiation." By the Hon J. W. Strutt, Fellow of Trinity College, Cambridge. Communicated by W. Spottiswoode, F.R.S. Received May 17, 1870.

(Abstract.)

These integrals present themselves in calculations dealing with arbitrary functions on the surface of a sphere which vary discontinuously in passing from one hemisphere to the other. When n, n' are both even or both odd, the values of the integrals may be immediately inferred from known theorems in which the integration extends from -1 to $+1$, or over the whole sphere; otherwise a special method is necessary. In the present paper a function of two variables is investigated, which, when expanded, has for coefficients the quantities in question. As an example of the method, the problem is taken of a uniform conducting sphere exposed to the heat proceeding from a radiant point. It will appear at once that the heat received by any element of the surface is expressed by different analytical functions on the two hemispheres—a source of discontinuity which renders necessary a special treatment of the problem. The solution is afterwards generalized to meet the case of a sphere exposed to any kind of radiation from a distance.

One remarkable result not confined to the sphere is, that the effect of a radiation which is expressed by one or more harmonic terms of odd order is altogether nil, with the single exception of the term of the first order.

XIII. "Note on the Construction of Thermopiles." By the Earl of Rosse, F.R.S. Received June 14, 1870.

Although in the measurement of small quantities of radiant heat by means of the thermopile much may be done towards increasing the sensibility of the apparatus by carefully adjusting the galvanometer and rendering the needle as nearly astatic as possible, there must necessarily be some limit to this, and it therefore appears desirable that the principles on which thermopiles of great sensibility can be constructed should also be carefully attended to.

With the view of obtaining a pair of thermopiles of greater sensibility and of more equal power than I had been able to procure ready made, I made a few experiments with various forms of that instrument, and I was led to the conclusion (one which might have been foreseen) that the

sensibility of the thermopile is much increased by reduction of its mass, and more especially by a diminution of the cross section of the elements.

To obtain a clear idea of the problem before us, which is, how to construct the thermopile so that, with a given amount of radiant heat falling on its face, the greatest current may be sent through the galvanometer, let us consider the thermopile under two different conditions:—

1. With the circuit open.
2. With the circuit complete.

In the first case, when radiant heat falls on the face of the pile, the whole mass of metal rises in temperature, the rise being greatest at the anterior face, and less and less as you approach the other end. This rise of temperature will increase till the heat radiated from the anterior face, together with that which traverses the depth of the pile and is radiated from the posterior face, is just equal to that radiated to the anterior face at that moment, or when

$$k(t+t')=kt+\frac{sc}{l}(t-t')=Q,$$

where (t, t') are respectively the temperatures of the anterior and posterior face, s, l the cross-section and depth of the pile, c proportional to the mean conductibility of the material of the pile, (Q) the quantity of heat falling on the pile in a unit of time, and (k) a constant.

Let us now suppose the circuit completed, and we shall have, in addition to the above, two causes operating to reduce the temperature of the anterior face,—the abstraction of heat *by* the electric current, and *proportional* to that current $=$LI, where I is the intensity of the current and L a constant, then there will be equilibrium when

$$k(t+t')+LI=kt+\frac{sc}{l}(t-t')+LI=Q.$$

It is quite clear therefore that if Q be constant, I will become the larger the smaller the other two terms become; and therefore as long as the first term continues small compared with the remaining terms, and the resistance in the pile is very small compared with that in the rest of the circuit, we shall increase the intensity of the current by every reduction of the cross-section of the elements of the thermopile.

There is another point which, though less important, cannot be entirely lost sight of, namely, that the more we reduce the mass of the anterior face and adjacent parts of the pile, the more rapidly will the temperature rise to its state of equilibrium, and therefore the more convenient will it be for use where the needle is liable to disturbances from various causes, and where consequently the more speedily the needle can be brought to rest, the more accurately will its observed motion be a measure of the radiant heat falling at that moment on the face of the pile.

Let us now compare the case of a single pair of small cross-section with a metal disk soldered to the junction of the two bars, and of suffi-

cient size to catch all the radiant heat required to be measured, with that of a pile of (n) pairs, each of equal dimensions with those of the single pair, the area of face being the same in the two cases.

By increasing the number of elements from one to n, we increase the number of solderings in that proportion; consequently the average amount of heat reaching any soldering is $\frac{1}{n}$ as great as that reaching the soldering of the single pair; therefore, if the same percentage of the total heat be lost by conduction, the total electromotive force is the same in the two cases. But inasmuch as the total cross-section of metal to conduct the heat away from the anterior face is n times as great in the pile as in the pair, and the resistance of the pile is n times as great as that of the pair, the pile will be inferior in power to the pair, unless these two causes of inferiority are counterbalanced by the loss due to the greater average distance to the soldering from the points where the heat reaches the face, in the case of the pair, than that of the pile of n pairs.

The experiments already referred to were made with three different thermoelectric pairs. These consisted each of a pair of bars of bismuth and an alloy of twelve parts of bismuth and one part of tin of different thicknesses, of about equal lengths in each case, and soldered about $\frac{1}{4}$ inch apart upright, on disks of sheet copper of $\frac{1}{2}$ inch diameter. A slip of wood was placed between the two bars, to protect them from injury, and to which they were fixed with thread. The three piles were compared with a pile of four elements, made by Messrs. Elliott, and the deviation due to the latter being taken equal to unity, the following deviations were obtained for the three thermo-pairs:—

	Weight of disk face.	Weight of two bars.	Deviation.	Metals employed.
I.	8 grains	42 grains	·676	Bismuth, antimony.
II.	$4\frac{1}{2}$,,	6 ,,	1·35	Bismuth $\left\{\begin{array}{c}\text{Bismuth,}\\ \text{tin } \frac{1}{12}.\end{array}\right\}$
III.	$\frac{1}{2}$ grain	3 ,,	3·23	Bismuth $\left\{\begin{array}{c}\text{Bismuth,}\\ \text{tin } \frac{1}{12}.\end{array}\right\}$

A heavy and a light pile were also compared, taking the interval between raising and depressing the screen, first $=\frac{1}{2}$ minute, and then $=2$ minutes; and it was found that, in the first case,

$$\frac{\text{Deviation due to light pair}}{\text{Deviation due to heavy pair}}=2{\cdot}6\,;$$

and, in the second case,

$$\frac{\text{Deviation due to light pair}}{\text{Deviation due to heavy pair}}=2{\cdot}9\,;$$

so that the light pair arrived rather more rapidly at the condition of equilibrium than the heavier pair.

Although the above experiments are far less complete than I could have wished, they are sufficient to show that the sensibility of thermopiles may be considerably increased by diminution of the section of the bars composing them; whether they may be with advantage reduced to a greater extent than I have already done I cannot say, but I am inclined to think that they may. I have ascertained from Messrs. Elliott that the alloys used by them in the construction of thermopiles, at the time when I received mine from them, were 32 parts of bismuth +1 part of antimony, and $14\frac{1}{7}$ of bismuth +1 part of tin. If allowance be made for the substitution of the first of these two alloys for pure bismuth, the difference between Elliott's pile and the pairs II. & III. will be rather greater. The pile by Messrs. Elliott, if made of the same metals as I employed, would have been reduced in power from 1 to 0·9.

The construction of thermo-couples, on the plan I have described, is comparatively easy. In about two hours I was able to make one, and in more experienced hands their construction would be still easier.

An experiment was made with one of the piles to ascertain whether, when the heat was not directed centrally on the pile, much diminution of power would take place. There *was* less deviation in consequence of the increase of the mean distance which the heat had to travel before it reached the soldering; but I believe that this defect might be remedied, probably without diminution of the power of the pile, by increasing the thickness of the face, and leaving the dimensions of the bars the same.